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Entropy parameters for heat exchanger design

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Abstract—A general expression for entropy generation in counter-current heat exchangers is developed. It is applicable to incompressible liquids and perfect gases. Two new entropy generation numbers are defined, N_M and N_Q. The analysis is applied to an air–air counter-current heat exchanger. The three entropy generation numbers, N_S, N_M and N_Q, have a different variation with NTU at the various values of the capacity flow rate ratio employed in the calculations.

1. INTRODUCTION

EVALUATION of irreversibilities in heat exchanger design has become generally accepted since the work of Bejan [1]. The initial limitations of the analysis, nearly ideal and nearly balanced capacity flow rate heat exchangers, allowed the determination of a minimum in the entropy generation number N_S, defined as $N_S = \Delta S/C_{\max}$.

A different definition of N_S [2], $N_S = \Delta S/C_{\min}$, made possible the appearance of a maximum in an irreversibility function not including friction losses.

Further studies on compact crossflow heat exchangers [3] and on regenerators of gas turbines [4] were mainly concerned with the optimization through the choice of the minimum entropy production.

This paper deals with:

(a) developing a general expression of the entropy generation;

(b) defining two new entropy generation numbers;

(c) investigating the relative position of both the maximum and minimum in the entropy generation numbers.

2. ENTROPY GENERATION

For the heat exchanger of Fig. 1 the entropy generation rate is given by

$$dS = m_1\,ds_1 + m_2\,ds_2 \tag{1}$$

where the heat transfer to the environment is neglected.

Expressing the entropy variation in a general way [5]

$$ds = c_p\,dT/T - (\partial v/\partial T)_p\,dp \tag{2}$$

the integration between inlet and outlet gives

$$\Delta S = m_1\left\{c_{p1}\ln(T_o/T_i)_1 - \int_i^o (\partial v/\partial T)_{1,p}\,dp_1\right\} + m_2\left\{c_{p2}\ln(T_o/T_i)_2 - \int_i^o (\partial v/\partial T)_{2,p}\,dp_2\right\} \tag{3}$$

with the assumption that c_{p1} and c_{p2} are averaged between T_i and T_o for each stream. For a liquid it is assumed that

$$(\partial v/\partial T)_p = \beta v = \text{const.} \tag{4}$$

while for a perfect gas

$$\int_i^o (\partial v/\partial T)_p\,dp = \int_i^o R/p\,dp = R\ln(p_o/p_i) = R\ln(1+\Delta p/p_i) \tag{5}$$

with the hypothesis $\Delta p/p_i \ll 1$ one gets

$$\int_i^o (\partial v/\partial T)_p\,dp = R\ln(1+\Delta p/p_i) \approx (R/p_i)\Delta p. \tag{6}$$

In general it is possible to express

$$\int_i^o (\partial v/\partial T)_p\,dp = l\Delta p \tag{7}$$

where

$$l = \beta v \tag{8}$$

for a liquid, and

$$l = R/p_i \tag{9}$$

for a perfect gas.

With the introduction of the efficiency ε [6], as

$$\varepsilon = C_1(T_{o1}-T_{i1})/[C_1(T_{i2}-T_{i1})] = C_2(T_{i2}-T_{o2})/[C_1(T_{i2}-T_{i1})] \tag{10}$$

where

$$C_1 = C_{\min} = c_{p1}m_1, \quad C_2 = C_{\max} = c_{p2}m_2 \tag{11}$$

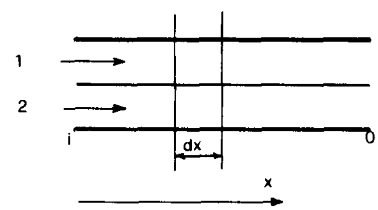

FIG. 1. Heat exchanger.

NOMENCLATURE

A	heat exchanger surface
B	stream section
C	heat capacity flow rate
c_p	specific heat
D_e	equivalent diameter
f	friction factor
k	thermal conductivity
h	convective heat transfer
L	heat exchanger length
m	mass flow rate
N_M, N_Q, N_S	entropy generation numbers
NTU	number of transfer units
Nu	Nusselt number, hD_e/k
p	pressure
Pr	Prandtl number, $\mu c_p/k$
r	temperature ratio
R	constant of gas
Re	Reynolds number, WD_e/v
Q	heat transferred in the exchanger
S	entropy rate
s	specific entropy
T	temperature
v	specific volume
z	capacity flow rate ratio.

Greek symbols

β	volumetric expansion coefficient
Δ	variation
ε	efficiency
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density.

Subscripts

1, 2	streams
i	inlet
max	maximum
min	minimum
o	outlet
Q	relative to the heat exchanger.

and

$$r = T_2/T_1\,; \quad z = C_1/C_2$$

equation (3) becomes

$$\Delta S = C_1\{\ln[1+\varepsilon(r-1)] + 1/z\ln[1-\varepsilon z(r-1)/r] - l_1\Delta p_1/c_{p1} - l_2\Delta p_2/(zc_{p2})\}. \quad (12)$$

With a modification of the logarithmic expression and using the entropy generation number as in ref. [2], one has

$$N_S = \Delta S/C_{\min} = N_{SP} + N_{SZ} + N_{S\varepsilon} \quad (13)$$

where

$$N_{SP} = -l_1\Delta p_1/c_{p1} - l_2\Delta p_2/(zc_{p2}) \quad (14)$$

$$N_{SZ} = \ln r + 1/z\ln[1-z(r-1)/r] \quad (15)$$

$$N_{S\varepsilon} = \ln[1-(r-1)(1-\varepsilon)/r] + 1/z\ln\{1+[z(r-1)(1-\varepsilon)/r]/[1-z(r-1)/r]\}. \quad (16)$$

It can be noted that N_{SZ} becomes zero for $z = 1$ and/or $r = 1$ while $N_{S\varepsilon}$ is zero for $\varepsilon = 1$ and/or $r = 1$. Further on, $N_{S\varepsilon}$ has a maximum for $\varepsilon = 1/(z+1)$, as found in ref. [2].

Neglecting the concentrated pressure losses and expressing the distributed ones as

$$\Delta p = 2f\,Re^2 v^2 L\rho/D_e^3 \quad (17)$$

it is possible to write

$$N_{SP} = 2[f_1\,Re_1^2 v_1^2 L_1\rho_1 l_1/(D_{e1}^3 c_{p1}) + f_2\,Re_2^2 v^2 L_2\rho_2 l_2/(zD_{e2}^3 c_{p2})]. \quad (18)$$

The entropy generation number proposed by Sarangi and Chowdhury [2], N_S, can be interpreted as the entropy production related to the heat transferred for one degree of temperature difference. We can obtain other significant parameters comparing entropy production to the greatest amount of producible entropy.

The total entropy generation, equation (12), can be related to the following entropy production:

$$\Delta S_Q = Q(1/T_{i1} - 1/T_{i2}) \quad (19)$$

giving

$$N_Q = \Delta S/\Delta S_Q = (N_S/\varepsilon)[r/(r-1)^2]. \quad (20)$$

The term ΔS_Q can be interpreted as the entropy generated if the heat transferred in the heat exchanger is exchanged between the inlet temperature of the two streams.

A maximum entropy generation can be defined by

$$\Delta S_{\max} = Q(1/T_{i1} - 1/T_{i2}) + m_1\Delta s_{1,\max} + m_2\Delta s_{2,\max} \quad (21)$$

where $\Delta s_{\max}$ is due to the free expansion of the fluid from the inlet pressure to zero. For a liquid

$$\Delta s_{\max} = l\Delta p_i \quad (22)$$

for a perfect gas the expansion can be carried on from the inlet pressure only to a very low pressure, p_o

$$\Delta s_{\max} = R\ln(p_i/p_o). \quad (23)$$

Then, the entropy generation number N_M can be introduced as

$$N_M = \Delta S/\Delta S_{\max} = \Delta S/[Q(1/T_{i1} - 1/T_{i2}) + m_1\Delta s_{1,\max} + m_2\Delta s_{2,\max}]. \quad (24)$$

3. APPLICATION TO AN AIR–AIR COUNTER-CURRENT HEAT EXCHANGER

For the heat exchanger of Fig. 2 the efficiency ε is given by [6]

$$\varepsilon = \{1-\exp[-NTU(1-z)]\}/\{1-z\exp[-NTU(1-z)]\} \quad (25)$$

where, neglecting the heat conduction in the wall

$$1/NTU = B_1/(A_1 St_1)+zB_2/(A_2 St_2). \quad (26)$$

According to equation (25), the maximum of $N_{S\varepsilon}$ for $\varepsilon = 1/(z+1)$ gives a maximum for $NTU = \ln\{1/[z(1-z)]\}$. The following relations are assumed valid for each stream [7]:

$$Nu = 0.023\Omega\, Re^{0.8}\, Pr^{0.33} \quad (27)$$

with

$$\Omega = 1+(D_e/L)^{0.7} \quad \text{if } L/D_e \leqslant 20 \quad (28)$$

and $\Omega = 1$ for $L/D_e > 20$

$$f = 0.079 Re^{-0.25} \quad \text{for } Re > 2200 \quad (29)$$

and

$$f = 16/Re \quad \text{if } Re \leqslant 2200. \quad (30)$$

The data of ref. [8] for air are interpolated with the least squares method by the following expressions:

$$c_p = 1013.3578-0.1645T+5.0824\times10^{-4}T^2-2.156\times10^{-7}T^3 \quad (31)$$

$$\mu = (3.9416+0.0521T-1.6449\times10^{-5}T^2+2.782\times10^{-9}T^3)\times10^{-6} \quad (32)$$

$$k = 1.898\times10^{-3}+8.9614\times10^{-5}T-3.7318\times10^{-8}T^2+9.9215\times10^{-12}T^3 \quad (33)$$

$$\rho = 2.6896-7.1084\times10^{-3}T+7.4569\times10^{-6}T^2-2.6636\times10^{-9}T^3 \quad (34)$$

where the units are in S.I. and T in Kelvin.

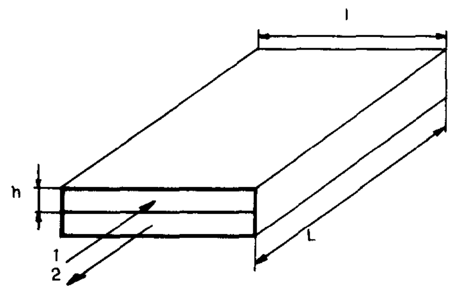

FIG. 2. Air–air counter-current heat exchanger.

The following calculations are obtained for a heat exchanger with surface $A = 1$ m^2, length $L = 1$ m, mass flow rate of air $m_1 = 1\times10^{-4}$ kg s^{-1}, $T_1 = 300$ K.

The components of N_S, as given in equation (13), are reported in Fig. 3 vs NTU, for $z = 0.999$ and 0.01. It can be noted that N_{SZ} is in general constant for fixed z and r and negligible for $z = 0.999$. On the other side, $z = 0.999$, $N_{S\varepsilon}$ has a maximum for $NTU = \ln[1/\{z(1-z)\}]$ and decreases with the increase in NTU. The contribution of N_{SP}, low at low NTU, increases with NTU. The resulting trend of N_S is different whether NTU is low or high. For low NTU, N_S presents a maximum which coincides with that of $N_{S\varepsilon}$, for greater NTU the contribution of N_{SP} is larger and for very large NTU the value of N_S is mainly given by N_{SP}.

At intermediate NTU a minimum of N_S is observable which is qualitatively comparable with that of ref. [1] for a simplified case. The values of N_S for $z = 0.01$ are greater than for $z = 0.999$.

Figure 4 presents the three-dimensional graph of N_S vs NTU and z. The maximum and minimum are less pronounced as far as z is decreased and for $z = 0.01$ they disappear. Further on, N_S increases with the decrease of z and for $z = 0.01$ it reaches a maximum value.

The number N_M is presented in Figs. 5 and 6 in similar two- and three-dimensional graphs. The trend

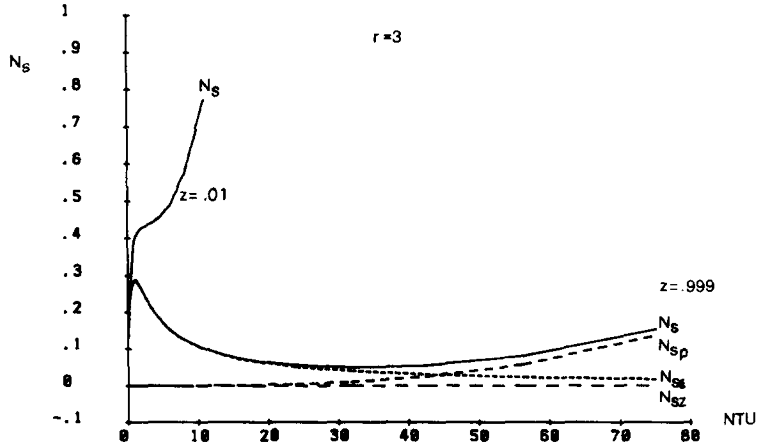

FIG. 3. N_S vs NTU.

of Fig. 5 is similar to that of Fig. 4 with the presence of both a maximum and minimum at high z values and their disappearance at low z. The new result of the calculations is the lower values of N_M for $z = 0.01$ up to $NTU \approx 20$. This conclusion seems to be due to the presence of m_2 in the denominator of equation (24).

The three-dimensional graph of N_M, Fig. 6, displays a very different picture than Fig. 4. The two numbers, N_S and N_M, have a similar trend up to about $z = 0.5$ while at low z values their trend is different. For NTU lower than 20, N_M decreases and it attains a minimum value at $z = 0.01$. When NTU is higher than 20 the minimum of N_M is obtained for $z = 0.999$.

The number N_Q is finally presented in Figs. 7 and 8 vs NTU. No relative maximum is observed either at $z = 0.999$ or 0.01. The relative minimum of N_Q at $z = 0.999$ is found at an NTU value comparable to that of N_S and N_M. At $z = 0.01$, N_Q presents a minimum and the values of N_Q are always higher than

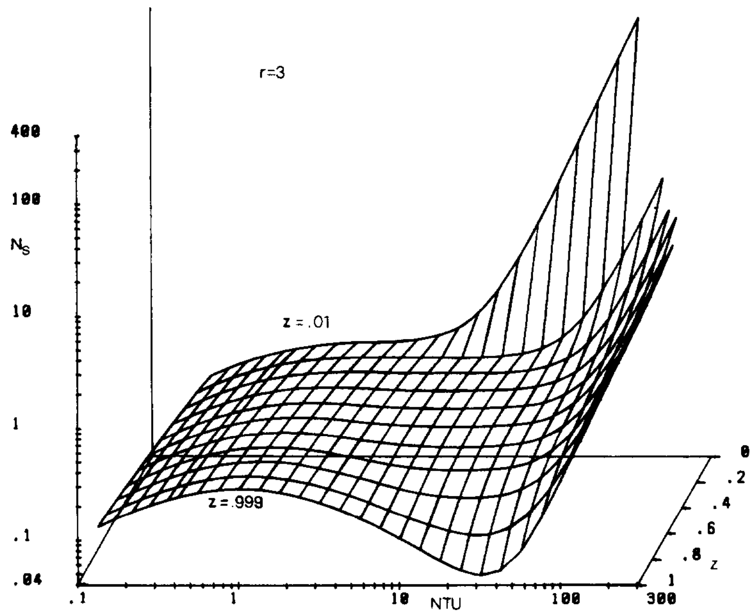

FIG. 4. Three-dimensional graph of N_S.

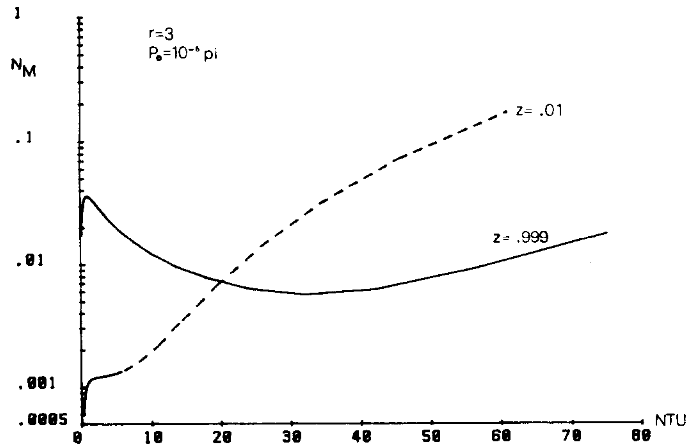

FIG. 5. N_M vs NTU.

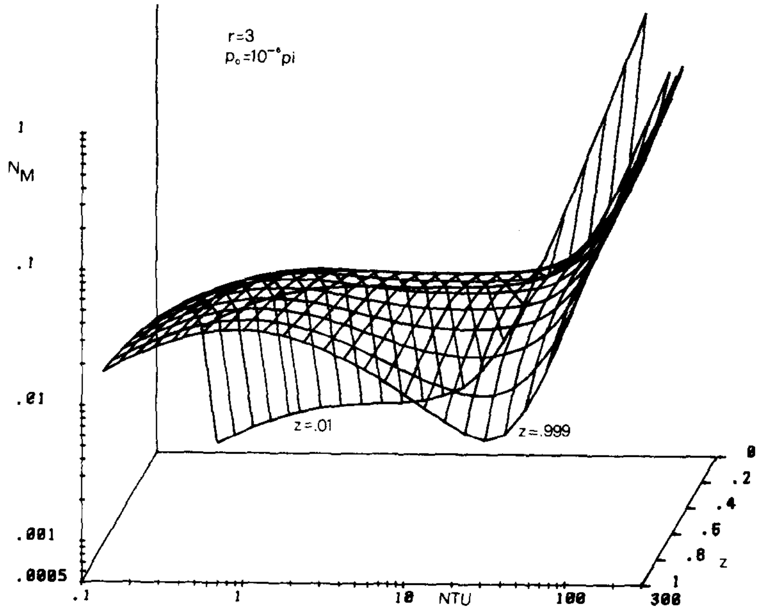

FIG. 6. Three-dimensional graph of N_M.

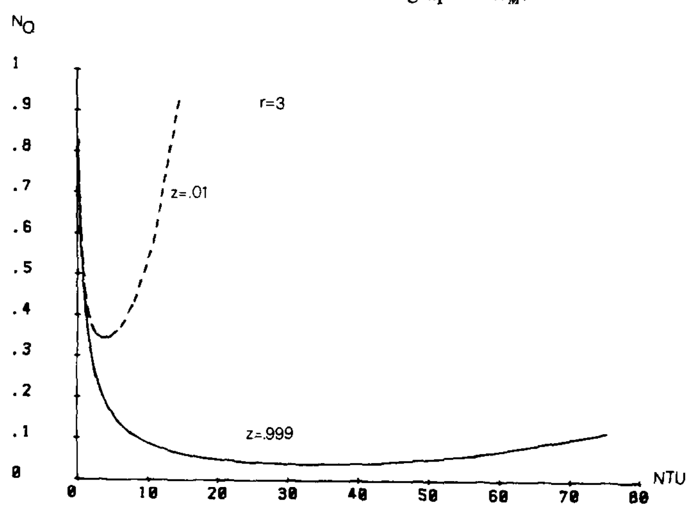

FIG. 7. N_Q vs NTU.

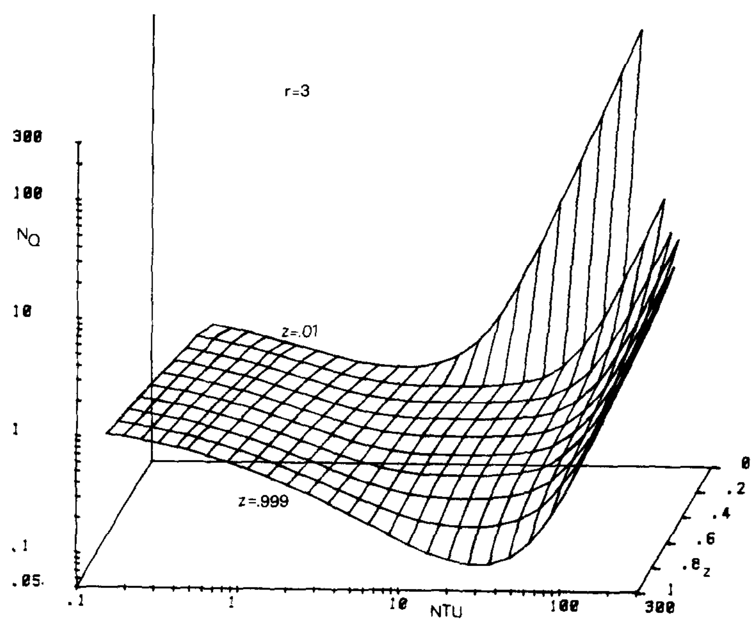

FIG. 8. Three-dimensional graph of N_Q.

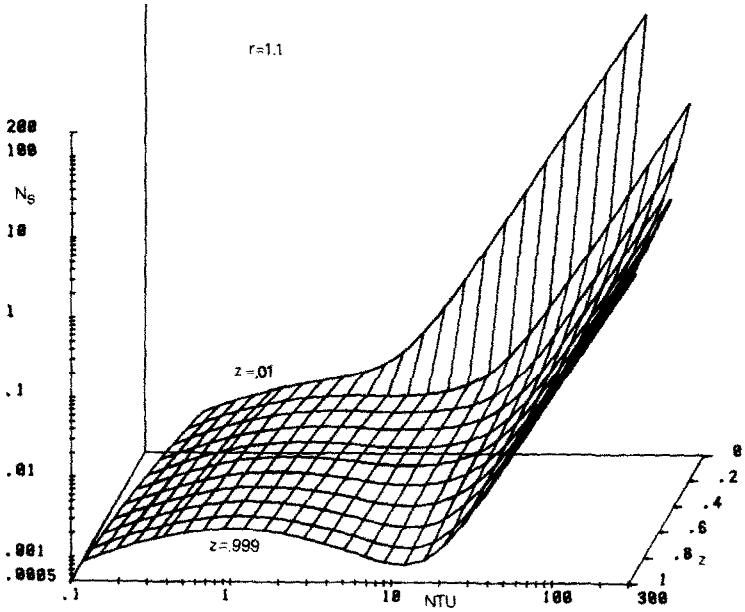

FIG. 9. Three-dimensional graph of N_S, $r = 1.1$.

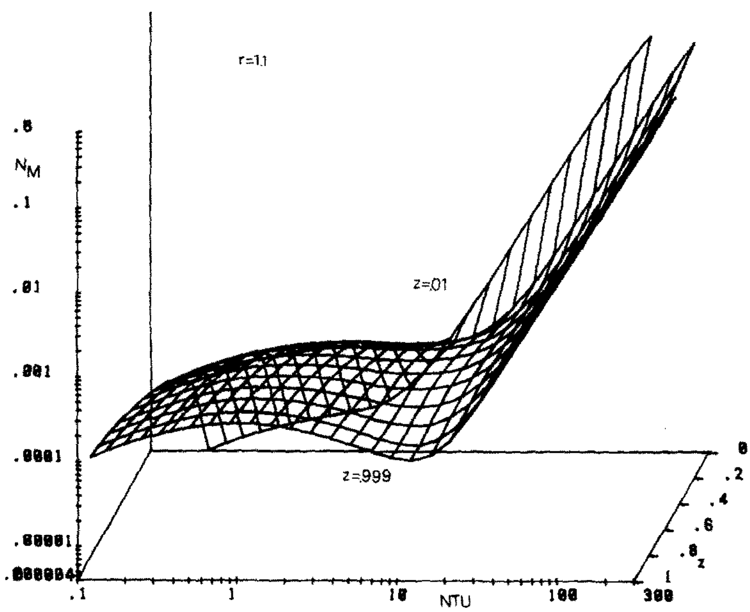

FIG. 10. Three-dimensional graph of N_M, $r = 1.1$.

those calculated at $z = 0.999$. The three-dimensional variation of N_Q is reported in Fig. 8. The trend of N_Q is to increase regularly from $z = 0.999$ to 0.01.

Figures 9–11 present the three entropy generation numbers, N_S, N_M and N_Q at $r = 1.1$. The relative minimum of the numbers at $z = 0.999$ is found for $NTU \approx 12$. The graphs are qualitatively similar to those obtained at $r = 3$, similar graphs can be obtained also for $r < 1$.

4. CONCLUSIONS

Entropy production for heat exchangers has been derived in a general way. The analysis includes incompressible fluids and perfect gases. Three entropy production numbers have been investigated N_S, N_M and N_Q; two of them (N_M and N_Q) newly defined. The application of the analysis to an air–air countercurrent heat exchanger has shown the following conclusions:

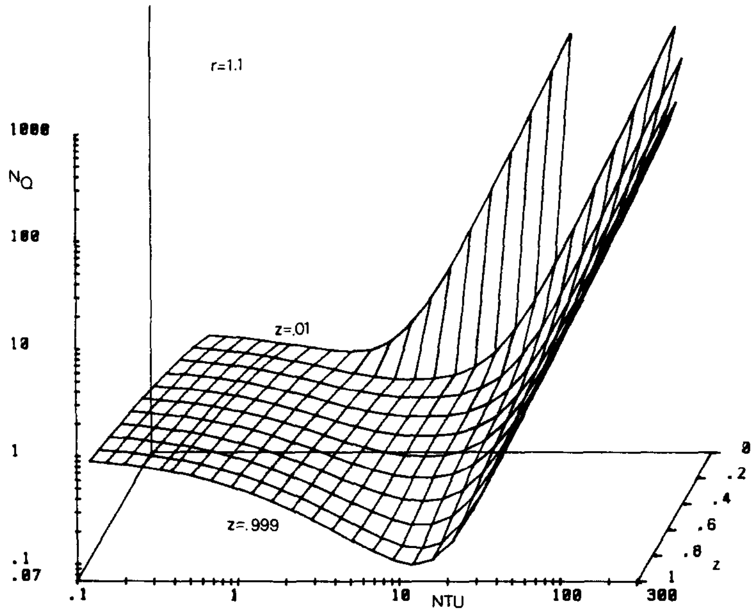

FIG. 11. Three-dimensional graph of N_Q, $r = 1.1$.

(a) N_S and N_M vs NTU have a maximum and minimum at $z = 0.999$ and no relative extremes at $z = 0.01$;

(b) N_S increases as z decreases;

(c) N_M has a relative maximum as a function of z;

(d) N_Q vs NTU does not present any maximum but only a minimum in the whole range of z;

(e) N_Q values increase as z is decreased.

Then N_Q might be utilized as an optimization criterion for heat exchanger design, taking into account entropy generation.

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PARAMETRES LIES A L'ENTROPIE POUR LA CONCEPTION DES ECHANGEURS DE CHALEUR

Résumé—On développe une expression générale pour la création d'entropie dans les échangeurs de chaleur à contre-courant. Elle est applicable aux liquides incompressibles et aux gaz parfaits. On définit deux nouveaux nombres de création d'entropie N_M et N_Q. L'analyse est appliquée à un échangeur air–air à contre-courant. Les trois nombres N_S, N_M et N_Q ont des variations différentes en fonction du NTU pour les valeurs des débits calorifiques considérés dans les calculs.

ENTROPIE-PARAMETER FÜR DIE AUSLEGUNG VON WÄRMEÜBERTRAGERN

Zusammenfassung—Es wird eine allgemeine Beziehung für die Entropieerzeugung in Gegenstrom-Wärmeübertragern entwickelt. Diese ist für inkompressible Flüssigkeiten und für ideale Gase anwendbar. Zwei neue Kennzahlen der Entropieerzeugung, N_M und N_Q, werden definiert. Die Berechnung wird auf einen Luft-Luft-Gegenstrom-Wärmeübertrager angewandt. Die drei Kennzahlen der Entropie-Erzeugung, N_S, N_M und N_Q verändern sich unterschiedlich mit NTU, abhängig vom Verhältnis der Wärmekapazitätsströme.

ПАРАМЕТРЫ ЭНТРОПИИ ДЛЯ РАСЧЕТА ТЕПЛООБМЕННИКА

Аннотация—Выведено общее выражение для производства энтропии в противоточных теплообменниках. Оно применяется для случаев несжимаемых жидкостей и идеальных газов. Найдены два новых числа производства энтропии, N_M и N_Q. Анализируется противоточный теплообменник, в котором в качестве обоих газов используется воздух. Три числа производства энтропии, N_S, N_M и N_Q, зависят различным образом от количества единиц переноса при различных величинах отношения расхода тепловой мощности, используемых при расчетах.